NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Discussion
Research Brief

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What Are Some Strategies For Facilitating Productive Classroom Discussions?

One area that has been given a great deal of attention in the mathematics education literature, particularly over the past 25 years, is classroom discourse. This is evident not only in the body of published articles but also in the many policy documents calling for more student talk in mathematics classrooms (see, e.g., NCTM's *Principles and Standards for School Mathematics* [NCTM, 2000] and the Common Core State Standards [NGA Center and CCSSO, 2010]). Although these documents often use different language to describe their communication standards, they are all based on the common assumption that students learn mathematics best when they are given opportunities to speak about mathematics using the language of mathematics. Discussion, which is promoted in all of the documents, can therefore provide students with opportunities to communicate mathematically.

Because many of us learned to teach through the "apprenticeship of observation" (Lortie, 1975) in traditional classrooms, calls to shift from recitation to discussion-based lessons can be challenging. Many teachers are understandably unsure and overwhelmed by the call to use rich tasks and to facilitate discussions in mathematics class (see, e.g., Ball, 1993; Chazan, 1993). Over the past 15 years, fortunately, the field has begun to tackle the problem of providing teachers with guidelines and tools to support the facilitation of productive classroom discussions. Nine strategies for facilitating productive discussions are listed below and are discussed in more detail throughout the remainder of the paper.

- Attend to the classroom culture
- Choose high-level mathematics tasks
- Anticipate strategies that students might use to solve the tasks and monitor their work
- Allow student thinking to shape discussions
- Examine and plan questions
- Be strategic about "telling" new information
- Explore incorrect solutions
- Select and sequence the ideas to be shared in the discussion
- Use Teacher Discourse Moves to move the mathematics forward
- Draw connections and summarize the discussion

Attend to the Classroom Culture

The Discourse Project was a five-year, professional development–based study aimed at understanding how mathematics teachers' attention to their classroom discourse could impact their beliefs and practice over time (see Herbel-Eisenmann & Cirillo, 2009). An important realization that teachers involved in the project had was that if they wanted to change the classroom culture by moving students toward a more open, student-centered discourse, they needed to invite their students to participate in this shift. For example, in a book chapter focused on her action research in the Discourse Project, middle school teacher Jean Krusi (2009) wrote about how she involved her students by asking them what makes a good classroom discussion. Together, Krusi and her students constructed a list of five norms for classroom discussion: "Everyone is listening; Everyone is involved; Everyone puts out ideas; No one is left out," and "Everyone is understanding—if not at the beginning, then by the end" (p. 121). Krusi found that, in addition to emphasizing these kinds of social norms, she also needed to mention mathematical norms, such as what counts as evidence in mathematics. As the school year came to a close, students commented that they were participating more compared to the beginning of the year, and that they thought that the discussions were fun.

This example from Krusi's class is consistent with other recommendations from the literature. For example, Chapin and O'Connor (2007) insist that the most critical condition that will support both language and mathematics development is for teachers to establish conditions for *respectful discourse*. Similar to Krusi's student-generated norms, Hiebert et al. (1997) proposed the following norms of the classroom culture: Tasks must be accessible to all students; every student must be heard; and every student must contribute. Discussion is most productive when these kinds of prerequisite conditions of respectful and equitable participation are established in advance (Chapin & O'Connor, 2007). As mentioned above, accessible, high level tasks are also a critical element of a good discussion.

Choose High-Level Mathematics Tasks

Stein et al. (2000) defined a mathematical task as a mathematical problem or set of problems that address a related mathematical idea or concept. The nature of mathematics tasks chosen by the teacher is a critical element to facilitating productive discussions for at least two important reasons. First, mathematics instruction is typically organized and orchestrated around instructional tasks. More specifically, delivery of content in mathematics classrooms tends to consist of working on tasks, activities, or problems. Second, the tasks with which students engage are a critical factor in what students learn about mathematics and how they learn it (Stein, Remillard, & Smith, 2007). The relationship between good tasks and good discussions is simple: If we want students to have interesting discussions, we need to give them something interesting to discuss. Activities with a "low floor" (i.e., mathematics knowledge prerequisites are kept to a minimum) and a "high ceiling" (i.e., mathematics activities can be extended to include complex ideas and relationships) tend to create mathematics experiences worth talking about (Gadanidis, Hughes, Scucuglia, & Tolley, 2009) and give more students an entry point into the discussion. Supporting productive discourse can be made easier if teachers work with mathematical tasks that allow for multiple strategies, connect core mathematical ideas, and are of interest to the students (Franke, Kazemi, & Battey, 2007).

Past research has shown that teachers can find it difficult to maintain the cognitive demand of high level tasks. For example, in their study, Stein et al. (1996) found that tasks that were set up to require a high level of cognitive demand tended to decline into less demanding student engagement more than half of the time that they were implemented. Teachers can work to maintain the cognitive demand of a task by investing time before the lesson in the recommendation described next.

Anticipate Strategies That Students Might Use to Solve the Tasks and Monitor Their Work

Teaching in a manner that productively makes use of students' ideas and strategies that are generated by high-level tasks is demanding. It requires knowledge of mathematics content, knowledge of student thinking, knowledge of pedagogical "moves" that a teacher can make to lead discussions, and the ability to rapidly apply all of these in specific circumstances (M. Smith & Stein, 2011). To support teachers in this endeavor, Smith and Stein suggested five practices that are intended to make student-centered instruction more manageable. This is done by moderating the degree of improvisation required from the teacher in the midst of a discussion. Rather than providing an instant fix for mathematics instruction, the five practices provide "a reliable process that teachers can depend on to gradually improve their classroom discussions over time" (Stein, Engle, Smith, & Hughes, 2008, p. 335). The first two of the five practices are *anticipating* students' solutions to a mathematics task and *monitoring* students' actual work on the task as they work in pairs or groups.

Anticipating requires considering the different ways the task might be solved. This includes anticipating factors such as how students might mathematically interpret a problem, the array of correct and incorrect strategies students might use to solve it, and how those strategies might relate to the goal of the lesson (M. Smith & Stein, 2011). Anticipating can support teachers' planning by helping them to consider, in advance, how they might respond to the work that students are likely to produce and how they can use those strategies to address the mathematics to be learned.

Monitoring, as described by M. Smith and Stein (2011), is attending to the thinking of students during the actual lesson as they work either individually or collectively on the task. This involves not only listening to students' discussions with their peers, but also observing what they are doing and keeping track of the approaches students are using. Monitoring can support teachers by allowing them to help students get ready for the classroom discussion (e.g., asking students to have an explanation prepared that uses mathematically precise language). It can also help teachers identify strategies that will advance the "collective reflection" (Cobb, Boufi, McClain, & Whitenack, 1997) of the classroom community and prepare for the end-of-class discussion (M. Smith & Stein, 2011). The remaining three of the five practices for orchestrating productive discussions (i.e., selecting, sequencing, and connecting) will be elaborated in later sections of this paper.

Allow Student Thinking to Shape Discussions

In his work on language use in the classroom, Nystrand (1997) argued that people learn not merely by being spoken (or written) to, but also by participating in the discussion about the ideas. This theory of learning is based on the Vygotskian (1978) notion that people learn through social interaction. Discussions can provide students with opportunities to learn by talking with their peers in small groups and by engaging in argumentation, justification, and reasoning in whole-class discussions. In discussion-oriented classrooms, students' responses inform the teacher questions and shape the course of the classroom talk. In particular, the teacher validates particular students' ideas by incorporating their responses into subsequent questions. This type of discourse is much less teacher-directed and predictable because it is "negotiated" and jointly determined by both teachers and students as teachers pick up on, elaborate, and question what students say (Nystrand, 1990, 1991). These kinds of interactions are often characterized by "authentic" questions, which are asked to get information (e.g., "Can you tell us how you decided the answer was 5?"), not to test what students know and do not know. The primary function of a discussion is to construct group knowledge (Bridges, 1987), and questions are the key to fruitful discussions. The research on questioning is vast; therefore only a brief overview is provided below.

Examine and Plan Questions

Examining one's own questions and questioning patterns is an important start when looking more closely at the classroom discourse (see, e.g., Herbel-Eisenmann & Cirillo, 2009). This examination alone, however, has not been shown to do enough to support teachers in facilitating productive discussions that "focus on mathematical meaning and relationships and make links between mathematical ideas and relationships" (M. Smith & Stein, 2011, p. 50). A single, well-formulated question can be sufficient for an hour's discussion (Dillon, 1983). However, many studies have shown that while teachers ask a lot of questions, these questions frequently call for specific factual answers, resulting in lower cognitive thought (Gall, 1984; Perrot, 2002). Some question-types open up discussion, while others are more "closed" (Ainley, 1987). For example, one type of question takes the form of part-sentences "left hovering in mid-air for the student to supply the missing word or phrase" (Ainley, 1987, p. 24). An example of this 'fill-in-the-blank' type of question is: "This polygon has three sides so we call it a ...?" This kind of question is closed, both because it relates to matters of established fact and because the teacher has one "right" answer in mind. On the other hand, it creates the illusion of participation and cooperative activity (Ainley, 1987).

Examples of well-formulated questions are: "What is the relationship between the solutions to a quadratic equation and its graph?" or "Why did you solve the quadratic equation to help you graph the parabola?" To answer to these types of questions, students need to provide more than just one-word answers because the answers are complex and require a deeper level of thinking to give complete answers. More open questions are often better for opening discussion and maximizing the chances of individuals to contribute to the discussion, yet such questions tend to be underused (J. Smith, 1986). It can be useful to plan not only tasks but also good questions in advance of the lesson (M. Smith & Stein, 2011), and to consider what questions we can ask to avoid too much "telling."

Be Strategic About "Telling" Information

In a series of papers titled *Arbitrary and Necessary*, Hewitt (1999, 2001a, 2001b) urged mathematics educators to consider teaching approaches that allow students to discover the necessary (e.g., that the ratio of a circle's circumference to its diameter is a constant number that is approximately 3.14), while only telling students that which is arbitrary (e.g., that this constant ratio of a circle's circumference to its diameter is denoted as pi (ϖ)). This distinction between what to tell versus what to allow students to discover goes against traditional teaching methods where teachers were typically the deliverers of all information, both arbitrary and necessary.

Lobato, Clarke, and Ellis (2005) pointed out several drawbacks to the "teaching as telling" practice. Telling is undesirable when it: (a) minimizes the opportunity to learn about students' ideas and strategies; (b) focuses only on the procedural aspects of mathematics; (c) positions the teacher (rather than the students) as arbiters of mathematical truth; (d) minimizes the cognitive engagement on the part of students; (e) communicates to students that there is only one solution path; and (f) represents premature closure of mathematical exploration (p. 103). As an alternative to telling, the authors put forth the strategy of *initiating*. Initiating includes but is not limited to the following actions:

- Summarizing student work in a manner that inserts new information into the conversation
- Providing information that students need in order to test their ideas or generate a counterexample
- Asking students what they think of a new strategy or idea (perhaps from a "hypothetical" student)
- Presenting a counterexample
- Engaging in Socratic questioning in an effort to introduce a new concept
- Presenting a new representation of the situation (e.g., a graph to accompany a table of values)

These strategies offer alternatives to directly telling students information so that the teacher can productively move the discussion forward. Another strategy involves allowing the students to share their ideas as the basis of the discussion. Sometimes even incorrect strategies are worth exploring.

Explore Incorrect Solutions

Rather than only allowing correct solutions and strategies to surface in discussions, many teachers have taken steps to reduce the stigma attached to being wrong, thus communicating to students that mistakes are part of the learning process (Staples & Colonis, 2007). Some researchers have found that exploring incorrect solutions can serve as a springboard for discussion. This can give a focus to the discussion and engage students in figuring out why an idea does or does not make sense (Bochicchio et al., 2009). This move has several benefits, including: addressing common misconceptions, refining student thinking, prompting metacognition, and engaging students in developing hypotheses (Bochicchio et al., 2009). Staples and Colonis (2007) found that, in collaborative discussions, it was rare for something to explicitly be identified as "wrong." Rather, students' ideas were treated as "works in progress," and the focus of the teacher's guidance was to help the student and the class extend the idea that had been presented and continue to develop a viable solution collaboratively. Purposefully selecting and sequencing the presentation of student ideas can be an effective way to organize a discussion of both incorrect and correct student solutions.

Select and Sequence the Ideas to Be Shared in the Discussion

One of the primary features of a discussion-based classroom is that, instead of doing virtually all of the talking, modeling, and explaining themselves, teachers must encourage and expect students to do so. To do this effectively, teachers need to organize students' participation (National Council of Teachers of Mathematics, 1991). After monitoring the work of students as they explore the task (described above), teachers can select and sequence the ideas to be shared in the discussion (M. Smith & Stein, 2011). Selecting involves deciding which particular students will share their work with the rest of the class to get "particular pieces of the mathematics on the table" (Lampert, 2001, p. 140). Selecting which solutions will be shared by particular students is guided by the mathematical goal for the lesson and by the teacher's assessment of how each contribution will contribute to that goal. Sequencing is deciding on what order the selected students should present their work. Teachers can maximize the chances that their mathematical goals for the discussion will be achieved by making purposeful choices about the order in which students' work is shared (M. Smith & Stein, 2011). Smith and Stein suggested that teachers can also benefit from a set of moves that will help them lead whole-class discussions. Specifically, they focused on a set of "talk moves" that can be used to support students as they share their thinking with one another in respectful and academically productive ways.

Use Teacher Discourse Moves

In Classroom Discussions, Chapin, O'Connor, and Anderson (2003, 2009) introduced five "productive talk moves," which they described as suggested actions that were found to be effective in "making progress toward achieving [their] instructional goal of supporting mathematical thinking and learning" (p. 11). This claim was based on data from their work in Project Challenge, an intervention project initially aimed to provide disadvantaged elementary and middle school students with a reform-based mathematics curriculum that focused on mathematical understanding, with a heavy emphasis on talk and communication about mathematics. A goal of using the talk moves was to increase the amount of high-quality, mathematically productive talk in classrooms.

Building on Chapin et al. (2003), Herbel-Eisenmann, Cirillo, and Steele expanded this earlier work through a five-year project aimed at supporting teachers' facilitation of classroom discourse through the design of a professional development curriculum program. The curriculum supports secondary mathematics teachers in becoming more purposeful about engaging students in mathematical explanations, argumentation, and justification. A modified set of talk moves serves as a centerpiece of the curriculum. This set of Teacher Discourse Moves (TDMs) is a tool that can help facilitate productive and powerful classroom discourse. As part of the curriculum's overarching goals, productive focuses on how discourse practices support students' access to mathematical content. Powerful refers to how classroom discourse supports students' developing identities as knowers and doers of mathematics. There are six TDMs (cf. the five talk moves), which are defined in such a way that highlights what is special about thinking and reasoning in mathematics class as opposed to any other subject area (Herbel-Eisenmann, Steele, & Cirillo, in press). These six moves are:

- Waiting (e.g., Can you put your hands down and give everyone a minute to think?)
- Inviting Student Participation (e.g., Let's hear what kinds of conjectures people wrote.)
- Revoicing (e.g., So what I think I hear you saying is that if there was only one point of intersection, it would have to be at the vertex. Have I got that right?)
- Asking Students to Revoice (e.g., Okay, can someone else say in their own words what they think Emma just said about the sum of two odd numbers?)
- Probing a Students' Thinking (e.g., Can you say more about how you decided that?)
- Creating Opportunities to Engage with Another's Reasoning (e.g., So what I'd like you to do now is use Nina's strategy to solve this other problem with a twelve-by-twelve grid.)

The six TDMs can be particularly productive and powerful when they are purposefully used in combination with each other (e.g., Asking Students to Revoice after Probing a Students' Thinking). These moves can be used in conjunction with the Five Practices introduced above.

Draw Connections and Summarize the Discussion

The first four of the five practices mentioned above (Anticipating, Monitoring, Selecting, and Sequencing) work to set up the discussion, whereas Connecting is primarily meant to occur during the discussion. Rather than having mathematical discussions that consist of separate presentations of different strategies and solutions, the goal is "to have student presentations build on one another to develop powerful mathematical ideas" (Smith & Stein, 2011, p. 11). The teacher supports students in drawing connections between their solutions and other solutions in the lesson. The discussion should come to an end with some kind of summary of the key mathematical ideas. The students ideally leave with "residue" from the lesson, which provides a way of talking about the understandings that remain when the activity is over (Hiebert et al., 1997).

Concluding Thoughts

In this brief summary, various guidelines and tools were presented to support teachers' efforts to facilitate productive discussions. It is important to recognize that this review only scratches the surface of a growing body of work. Several important areas of this research could not be included here due to space. Some examples include: the teacher's role in classroom discourse (Walshaw & Anthony, 2008); the role of students (Hiebert et al., 1997); the development of mathematical language (see, e.g., Herbel-Eisenmann, 2002; Pimm, 1987); developing lesson goals and planning for productive discussions (Smith & Stein, 2011); using discussion as a formative assessment tool (Lee, 2006); types of questions (e.g., Boaler & Humphreys, 2005) and patterns of questioning (Herbel-Eisenmann & Breyfogle, 2005); equitable participation in classroom discussions (Esmonde, 2009); student motivation to participate in discussions (Jansen, 2006), and so on. There is still much to learn about the conditions under which discussions are productive toward reaching learning goals in mathematics classrooms. The guidelines and tools presented here, however, are intended to provide teachers with a place to begin working on their own goals of facilitating productive and powerful mathematics discussions.

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